EEE202 Fall 2024 EXAM #2 Fri 12-6-24

EXAM RULES

1) One 8.5" x 11" sheet permitted;
   otherwise closed books, closed notes, open minds!

2) Scientific calculator is permitted; no computers and no phone!

3) **NO PHONES !!!**
   Phones should not be visible at all!
   A visible phone will result in a zero grade for the exam!

4) Write FULL NAME legibly on each page provided.

5) **PLEASE SHOW ALL WORK !!!**
   This is essential to receive partial credit!

6) Please do not submit multiple answers. You must pick an answer!!

7) Write your solutions on the sheets provided.
   No other paper/sheets/pages should be used!

8) Clearly label voltages and currents on the circuits provided.

9) Use the variables provided!
   No additional variables should be used!

10) Unreadable work will receive NO CREDIT.

11) Please place important equations and answers within boxes as we have done in lecture.

12) Please be careful with your algebra, signs, etc.

13) Please turn in your solutions to me at the end of the period!

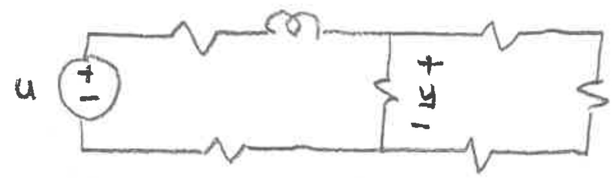
14) **PLEASE DO NOT CHEAT !!!**

From the desk of Dr. A.A. Rodriguez

Problem # 1

Determine H, diffeq, t_s, y_{ss}, y, Plot $|H(j\omega)|$, $\angle H(j\omega)$

(All $R, L = 1$)

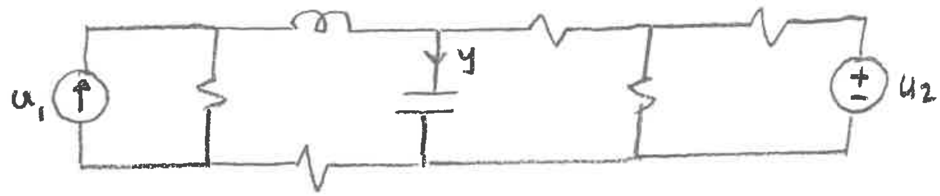


$u = 11 + 11\sin\left(\frac{11}{4}t\right)$

Problem # 1

Problem # 2

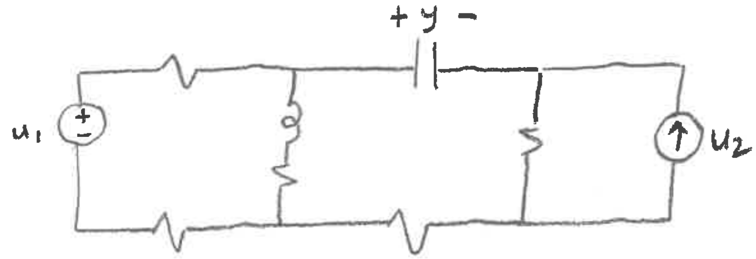
Determine poles, t_s, $H_i(0)$, $H_i(\infty)$ $(i=1,2)$ (All $R, L, C = 1$)



Problem #2

Problem # 3

Determine $H_{1,2}$, diff eq, t_s, y_{ss} (All $R, L, C = 1$)



$u_1 = 10 - \sin(t + 20°)$

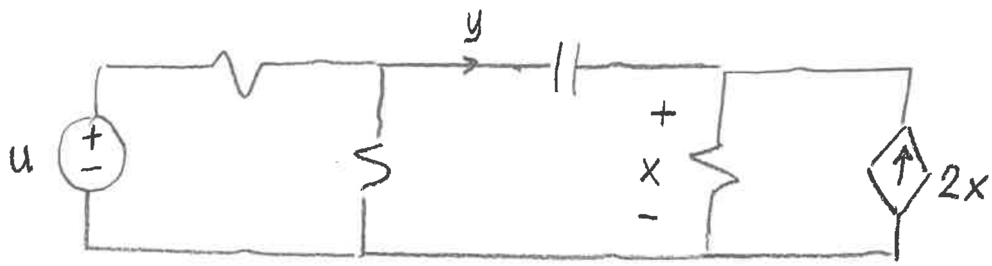
$u_2 = -3 + \cos(100t - 90°)$

Problem # 3

Problem #4

Determine H, diffeq, y, yss

(All R, C = 1)



$u = 4 + \sin 2t$

Problem #4

Problem # 5

Determine t_s, y_{ss}, y

Note: Show how to compute all coefficients associated with constants, sinusoids, & instabilities in y

$$H(s) = \frac{(s+0.01)(s^2+4)(s^2+64)}{(s+1)(s+2)(s^2+s+1)(s^2-4s+8)}$$

$$u = -10 + 2\cos(0.01t - 45^\circ) + 3\sin(2t + 20^\circ) - 4\sin(8t - 24^\circ) + 5\cos(100t + 90^\circ)$$

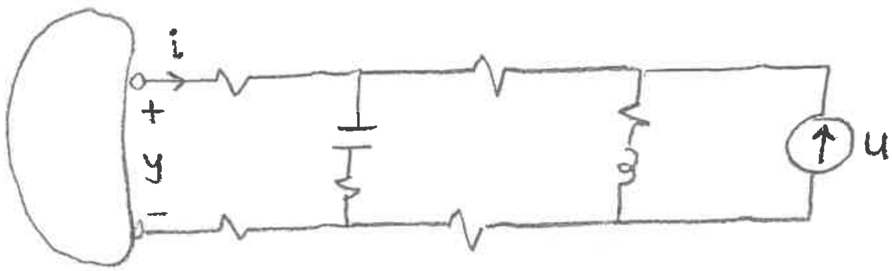
Problem # 5

(All R, L, C = 1)

Problem # 6

Determine an s-domain Thevenin equivalent at y

(specify Z_{th} & V_{th} --- emphasize process ... not algebra!)



Problem #6

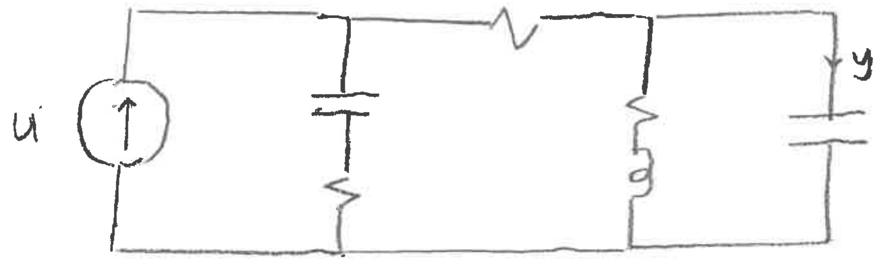
Problem #7

Determine H, y_{ss}, t_s

Compute all coefficients in y_{ss} (approximately)!

(all $R, L, C = 1$)

Hint: poles are
$-0.3522, -0.8239 \pm j0.8607$

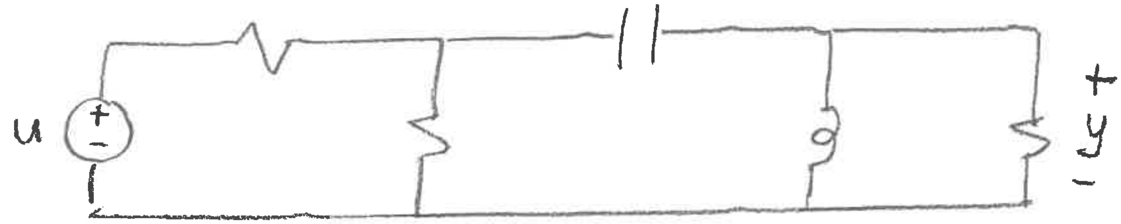


$u = -7 + 100\sin(0.01t - 90^\circ) - 2\cos(100t - 5^\circ)$

Problem #7

Problem # 8

Determine & plot $|H(j\omega)|$ & $\angle H(j\omega)$ (All R, L, C = 1)



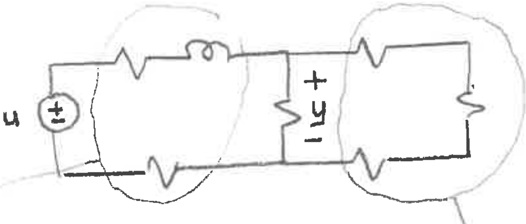
Problem # 8

Problem #1

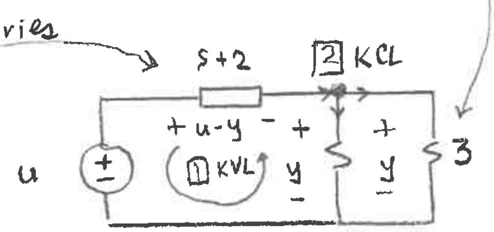
Determine H, diffeq, t_s, y_{ss}, y, $|H(j\omega)|$, $\angle H(j\omega)$ plot

(All $R, L = 1$)

$u(t) = 11 + 11 \sin\left(\frac{11}{4} t\right)$



Solution Via Series combinations, KVL, KCL, Ohm



$\boxed{2}$ KCL $=$ $\boxed{\left(\frac{u-y}{s+2}\right) = (y) + \left(\frac{y}{3}\right)}$

$$\Rightarrow y\left[\frac{1}{s+2} + \underbrace{\left(1 + \frac{1}{3}\right)}_{4/3}\right] = \frac{u}{s+2}$$

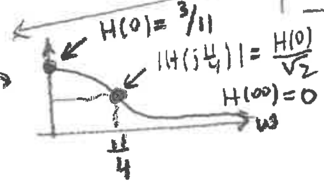
$$\Rightarrow y\left[1 + \frac{4}{3}(s+2)\right] = u$$

$$\Rightarrow y\left[\frac{3}{4} + s + 2\right] = \frac{3}{4} u$$

$$\Rightarrow \boxed{H = \frac{y}{u} = \frac{\frac{3}{4}}{s + \frac{11}{4}}}$$

$$H(j\omega) = \frac{\frac{3}{4}}{j\omega + \frac{11}{4}}$$

$$|H(j\omega)| = \frac{\frac{3}{4}}{\sqrt{\omega^2 + \left(\frac{11}{4}\right)^2}}$$



$$\dot{y} + \frac{11}{4} y = \frac{3}{4} u$$

$$\Phi = s + \frac{11}{4} = 0 \Rightarrow \text{pole at } s = -\frac{11}{4}$$

pole in LHP $\Longrightarrow$ system stable

$$t_s = \frac{5}{|\text{pole}|} = \frac{5}{\frac{11}{4}} = \frac{20}{11} \text{ sec}$$

$$\angle H(j\omega) = \angle top - \angle bottom = 0 - \tan^{-1}\left(\frac{\omega}{\frac{11}{4}}\right)$$

MOTF

$$\boxed{y_{ss} = 11\, H(0) + 11\, |H(j\tfrac{11}{4})| \sin\left(\tfrac{11}{4} t + \angle H(j\tfrac{11}{4})\right)}$$

$$H(0) = \frac{3}{11} \qquad H(j\tfrac{11}{4}) = \frac{\frac{3}{4}}{j\frac{11}{4} + \frac{11}{4}} = \frac{3/11}{\underbrace{1+j1}_{\sqrt{2}e^{j45}}} = \frac{3}{11\sqrt{2}} e^{j(-45^\circ)}$$

Find y

$$U(s) = \frac{11}{s} + 11\left[\frac{11/4}{s^2 + \left(\frac{11}{4}\right)^2}\right] = \frac{\overbrace{11\left(s^2 + \left(\frac{11}{4}\right)^2\right) + 11\left(\frac{11}{4}\right)s}^{\text{num}}}{s\left(s^2 + \left(\frac{11}{4}\right)^2\right)}$$

$$Y(s) = H(s)U(s) = \left(\frac{\frac{3}{4}}{s + \frac{11}{4}}\right)\left(\frac{\text{num}}{s\left(s^2 + \left(\frac{11}{4}\right)^2\right)}\right) = \frac{A}{s} + \frac{B}{s + \frac{11}{4}} + \frac{C}{s - j\frac{11}{4}} + *$$

$$\Rightarrow y = A + Be^{-\frac{11}{4}t} + \underbrace{2|C| \cos\left(\tfrac{11}{4} t + \angle C\right)}_{2|C| \sin\left(\frac{11}{4}t + \angle C + 90^\circ\right)}$$

y_{ss}

$$\Rightarrow A = \lim_{s \to 0} s Y(s) = 11\, H(0) \qquad B = \lim_{s \to -\frac{11}{4}} \left(s + \tfrac{11}{4}\right) Y(s)$$

$$C = \lim_{s \to j\frac{11}{4}} \left(s - j\tfrac{11}{4}\right) Y(s) = \frac{11 |H(j\frac{11}{4})|}{2} e^{j\left(\angle H(j\frac{11}{4}) - 90^\circ\right)}$$

Problem # 1

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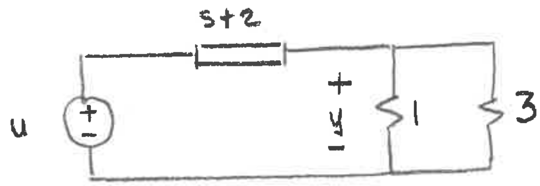
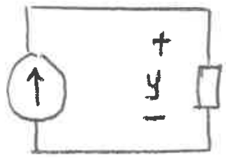
Solution via series - parallel & source transf



$\frac{u}{s+2}$



$$(s+2)\|(1)\|(3) = (s+2)\| \tfrac{3}{4} = \frac{\frac{3}{4}(s+2)}{s + \underbrace{2+\frac{3}{4}}_{\frac{11}{4}}}$$

$$\Rightarrow \quad y = \left[\frac{\frac{3}{4}\cancel{(s+2)}}{s+\frac{11}{4}} \right] \frac{u}{\cancel{s+2}}$$

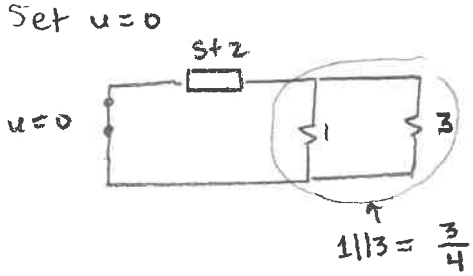
$$\Rightarrow \quad H(s) = \frac{y}{u} = \frac{\frac{3}{4}}{s+\frac{11}{4}}$$

☺

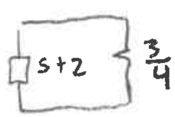
... just as before!

Pole Check (No KVL, KCL, Ohm)

Set $u = 0$



$\Rightarrow$



$$\Rightarrow \quad Z = s + 2 + \tfrac{3}{4}$$

$$= \underbrace{s + \tfrac{11}{4}}_{\Phi \text{ (characteristic eq)}}$$

$$= 0 \quad \Rightarrow \text{pole} = -\tfrac{11}{4} \checkmark$$

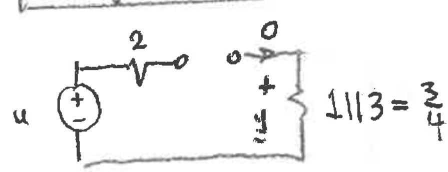
☺

Analysis at $s = 0$

$$y = \left(\frac{3/4}{\underbrace{2+3/4}_{11/4}}\right) u = \left(\frac{3}{11}\right) u$$

$H(0) = \frac{3}{11}$ ✓

Analysis at $s = \infty$



$\Rightarrow y = 0$

$\Rightarrow H(\infty) = 0$ ✓

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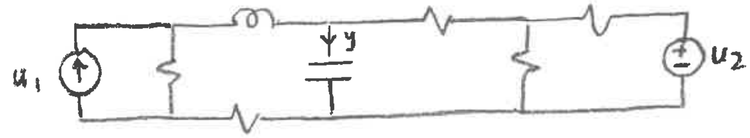
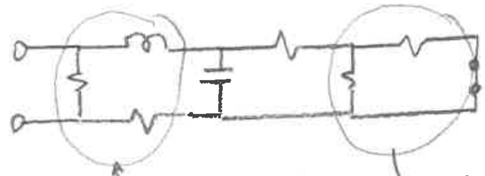
Problem #2

Determine poles, ts, $H_i(0)$, $H_i(\infty)$ (All R=L=C=1)

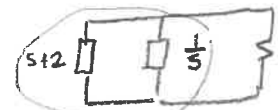


poles, ts

Set $u_1=0$, $u_2=0$



$\Longrightarrow$



$(s+2)\|\frac{1}{s} = \dfrac{\frac{s+2}{s}}{s+2+\frac{1}{s}} = \dfrac{s+2}{s^2+2s+1}$

$$Z = \frac{s+2}{s^2+2s+1} + \frac{3}{2}$$

$$= \frac{s+2+\frac{3}{2}(s^2+2s+1)}{s^2+2s+1}$$

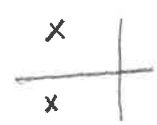
$$= \frac{\frac{3}{2}\left[\frac{2}{3}s+\frac{4}{3}+s^2+2s+1\right]}{s^2+2s+1} \qquad s^2+\frac{8}{3}s+\frac{7}{3}$$

$$\Phi = s^2+\frac{8}{3}s+\frac{7}{3}=0$$

$$s_{1,2} = \frac{-\frac{8}{3} \pm \sqrt{\frac{64}{9}-\frac{28}{3}}}{2(1)}$$

$$= -\frac{8}{6} \pm \sqrt{\frac{16}{9}-\frac{7}{3}} \qquad \frac{21}{9}$$

$$= -\frac{4}{3} \pm j\frac{\sqrt{5}}{3}$$



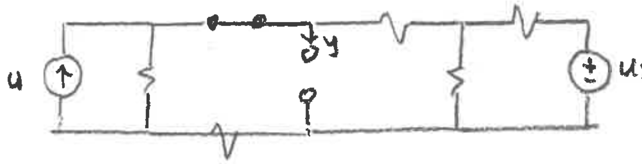
stable

$$t_s = \frac{5}{|\text{Re poles}|} = \frac{5}{\frac{4}{3}} = \frac{15}{4} \text{ sec}$$

Analysis at s=0

$Z_L = sL = 0$ (short)

$Z_C = \frac{1}{sC} = \infty$ (open)



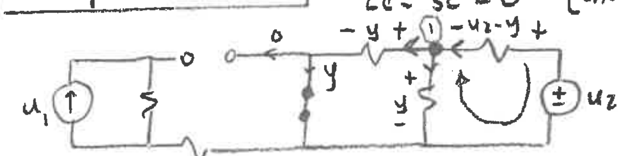
$\Longrightarrow y = 0 = 0u_1 + 0u_2$

$\Longrightarrow \boxed{H_1(0) = H_2(0) = 0}$

Analysis at s=∞

$Z_L = sL = \infty$ (open)

$Z_C = \frac{1}{sC} = 0$ (short)



u_1 has no impact on y! (only u_2 does!)

(1) KCL = $u_2 - y = y + y$

$\Longrightarrow 3y = u_2$

$\Longrightarrow y = 0u_1 + \left(\frac{1}{3}\right)u_2$

$\boxed{H_1(\infty)=0 \quad H_2(\infty)=\frac{1}{3}}$

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Problem #2

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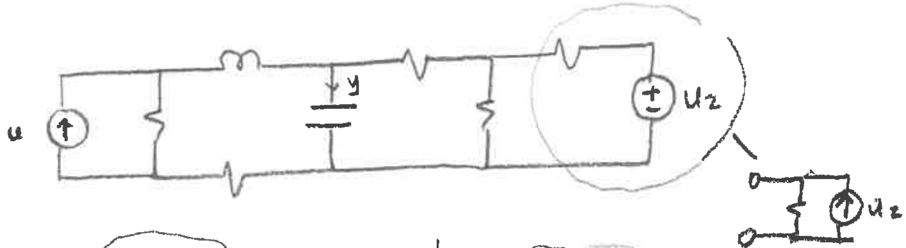
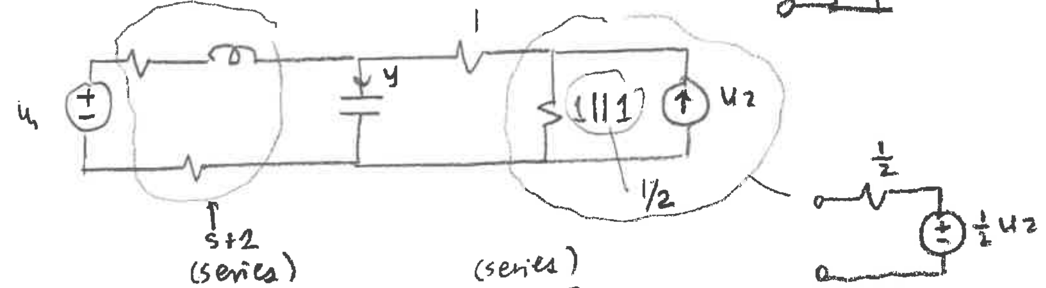
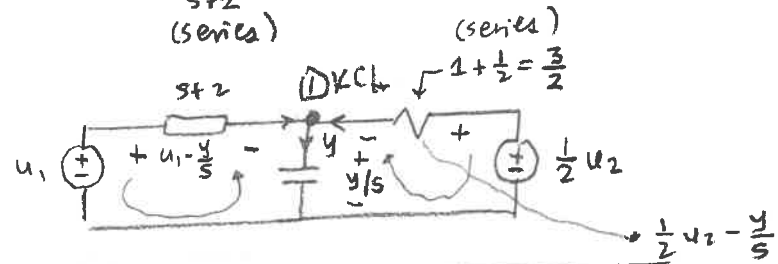
Solution via source transformations (All R, L, C = 1)







VERY IMPORTANT!

Note: Without source transformations or some other trick (e.g. thevenin equiv), we would NEED to INTRODUCE ANOTHER VARIABLE! (to solve the problem)

$$\boxed{(1)\ \text{KCL}: \left(\frac{u_1 - \frac{y}{s}}{s+2}\right) + \left(\frac{\frac{1}{2}u_2 - \frac{y}{s}}{\frac{3}{2}}\right) = y}$$

$$\Rightarrow y\left[\frac{1/s}{s+2} + \frac{2/3}{s} + 1\right] = \frac{u_1}{s+2} + \frac{u_2}{3}$$

$$\overset{s(s+2)\times}{\Rightarrow} y\left[\underbrace{1 + \tfrac{2}{3}(s+2) + s(s+2)}_{s^2 + (2+\frac{2}{3})s + 1 + \frac{4}{3} \;=\; s^2 + \frac{8}{3}s + \frac{7}{3}}\right] = s\,u_1 + \tfrac{1}{3}s(s+2)\,u_2$$

$$\boxed{y = \underbrace{\left[\frac{s}{s^2 + \frac{8}{3}s + \frac{7}{3}}\right]}_{H_1} u_1 + \underbrace{\left[\frac{\frac{1}{3}s(s+2)}{s^2 + \frac{8}{3}s + \frac{7}{3}}\right]}_{H_2} u_2}$$

Note: Same characteristic eq as before! --- $\Phi(s) = s^2 + \frac{8}{3}s + \frac{7}{3}$

$H_1(0) = H_2(0) = 0$ --- same as before!

$H_1(\infty) = 0 \quad H_2(\infty) = \frac{1}{3}$ --- same as before!

Problem #3

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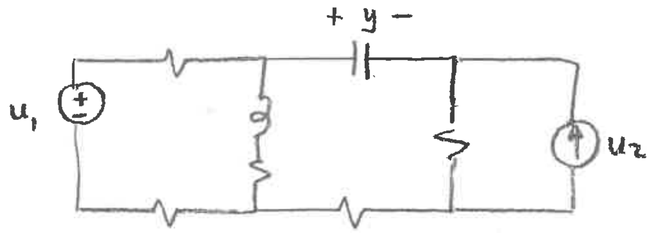
Determine $H_{1,2}$, diff eq, t_s, y_{ss} (All $R, L, C = 1$)



$u_1 = 10 - \sin(t + 20°)$

$u_2 = -3 + \cos(100t - 90°)$

solution:

Can 1st determine y_{ss} via MOTF!

(we know circuit will be stable since it consists of R, L, Cs & indep sources

... for a circuit to be unstable it is necessary to have dependent sources!!!)

Necessary but NOT sufficient!

linearity (superposition)

$y_{ss} = y_{1ss} + y_{2ss}$

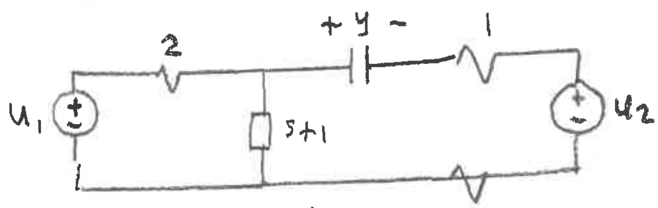
MOTF

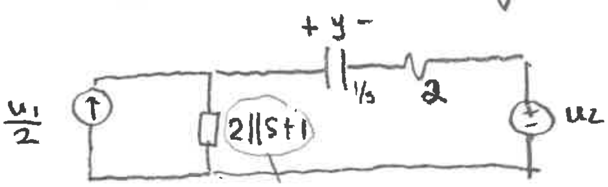
$y_{1ss} = 10 H_1(0) - |H_1(j1)| \sin(t + 20° + \angle H_1(j1))$

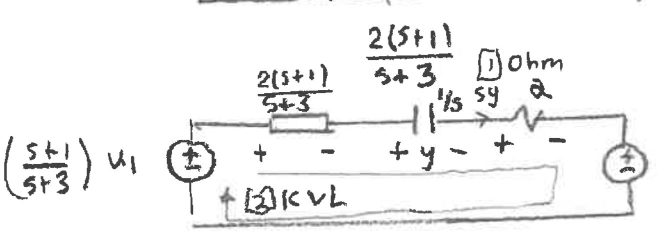
MOTF

$y_{2ss} = -3 H_2(0) + |H_2(j100)| \cos(100t - 90° + \angle H_2(j100))$

solution via series-parallel & source transformations







$\boxed{2}$ KVL $=$ $\boxed{\left(\frac{s+1}{s+3}\right)u_1 = \left[\frac{2(s+1)}{s+3}\right]sy + y + 2sy + u_2}$

$\Rightarrow y\left[\frac{2s(s+1)}{s+3} + 1 + 2s\right] = \left[\frac{s+1}{s+3}\right]u_1 - u_2$

$\overset{(s+3)*}{\Longrightarrow} y\left[2s(s+1) + s + 3 + 2s^2 + 6s\right] = (s+1)u_1 - (s+3)u_2$

$4s^2 + 9s + 3 = 4\left[s^2 + \frac{9}{4}s + \frac{3}{4}\right]$

$$\boxed{y = \left[\frac{\frac{1}{4}(s+1)}{s^2 + \frac{9}{4}s + \frac{3}{4}}\right]u_1 + \left[\frac{-\frac{1}{4}(s+3)}{s^2 + \frac{9}{4}s + \frac{3}{4}}\right]u_2}$$

H_1 H_2

diff eq =

$\ddot{y} + \frac{9}{4}\dot{y} + \frac{3}{4}y = \frac{1}{4}(\dot{u}_1 + u_1)$
$- \frac{1}{4}(\dot{u}_2 + 3u_2)$

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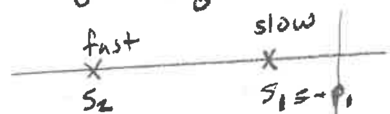
Problem # 3

poles

$$\Phi = s^2 + \frac{9}{4}s + \frac{3}{4} = 0 \Rightarrow s_{1,2} = \frac{-\frac{9}{4} \pm \sqrt{\frac{81}{16} - 3}}{2}$$

$$= -\frac{9}{8} \pm \sqrt{\frac{81}{64} - \frac{3}{4}} \qquad \frac{48}{64} \qquad \begin{array}{r} 81 \\ -48 \\ \hline 33 \end{array}$$

$$= -\frac{9}{8} \pm \frac{\sqrt{33}}{8}$$



stable system since all poles lie in LHP!

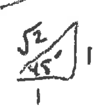
$$t_s = \frac{5}{|-s_1|} = \frac{5}{\underbrace{\frac{9}{8} - \frac{\sqrt{33}}{8}}_{p_1}}$$

Gains for y_{1ss}, y_{2ss}:

$$H_1 = \frac{\frac{1}{4}(s+1)}{s^2 + \frac{9}{4}s + \frac{3}{4}}$$

Needed in y_{1ss}

$$H_1(0) = \frac{1/4}{3/4} = \frac{1}{3}$$



$$H_1(j1) = \frac{\frac{1}{4}(j1+1)}{\underbrace{(\frac{3}{4} - 1)}_{-1/4} + j\frac{9}{4}} = \frac{\frac{1}{4}\sqrt{2}\, e^{j45°}}{\frac{1}{4}\sqrt{9^2+1}\; e^{j(180 - \mathrm{Tan}^{-1}(\frac{9}{1}))}} = \underbrace{\frac{\sqrt{2}}{\sqrt{82}}}_{|H_1(j1)|} e^{j\underbrace{(45° - 180 + \mathrm{Tan}^{-1}(9))}_{\angle H_1(j1)}}$$

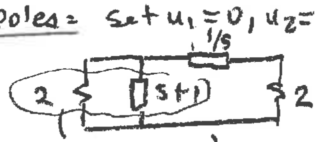
$$H_2 = \frac{-\frac{1}{4}(s+3)}{s^2 + \frac{9}{4}s + \frac{3}{4}}$$

Needed in y_{2ss}

$$H_2(0) = \frac{-1/4}{3/4} = -\frac{1}{3}$$

$$H_2(j100) \cong \frac{-1/4}{s}\Big|_{s = 100e^{j90°}} = \frac{\frac{1}{4}e^{j180°}}{100\, e^{j90°}} = \underbrace{\frac{1}{400}}_{|H_2(j100)|} e^{j\underbrace{(90°)}_{\angle H_2(j100)}}$$

check poles: set $u_1 = 0$, $u_2 = 0$



$$2 \| s+1 = \frac{2(s+1)}{s+3}$$

$$\Rightarrow Z = \frac{2(s+1)}{s+3} + \frac{1}{s} + 2 = \frac{2s^2 + 2s + s + 3 + 2s^2 + 6s}{s(s+3)}$$

$$4s^2 + 9s + 3 = 4\left[s^2 + \frac{9}{4}s + \frac{3}{4}\right]$$

$$\Rightarrow \Phi = s^2 + \frac{9}{4}s + \frac{3}{4} \quad \text{(same as before!)}$$

... hence same poles & t_s!

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Problem # 3

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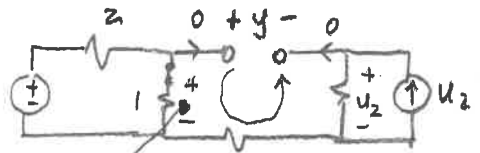
Check gains:

Analysis at $s = 0$

$Z_L = sL = 0$ (short)
$Z_C = \frac{1}{sC} = \infty$ (open)



voltage division: $\left(\frac{1}{2+1}\right) u_1 = \frac{u_1}{3}$

KVL

$\Rightarrow y = \left(\frac{1}{3}\right) u_1 + (-1) u_2$

$\boxed{H_1(0) = \frac{1}{3} \quad H_2(0) = -1}$ ← Gains Agree with

$$H_1 = \frac{\frac{1}{4}(s+1)}{s^2 + \frac{4}{4}s + \frac{3}{4}}$$

$$H_2 = -\frac{\frac{1}{4}(s+3)}{s^2 + \frac{4}{4}s + \frac{3}{4}}$$

Analysis at $s = \infty$

$Z_L = sL = \infty$ (open)
$Z_C = \frac{1}{sC} = 0$ (short)

+ y −

$\Rightarrow y = 0 u_1 + 0 u_2$

$\boxed{H_1(\infty) = H_2(\infty) = 0}$ ← Gains Agree with

Lets find form of y ... for the hell of it! ☺

$$U_1 = \frac{10}{s} + \frac{\quad}{s^2+1} = \frac{\text{num1}}{s(s^2+1)} \qquad U_2 = \frac{-3}{s} + \frac{\quad}{s^2+104} = \frac{\text{num2}}{s(s^2+104)}$$

$$Y = \overbrace{H_1 U_1}^{Y_1} + \overbrace{H_2 U_2}^{Y_2} = \left[\frac{\frac{1}{4}(s+1)}{(s+p_1)(s+p_2)}\right]\left[\frac{\text{num1}}{s(s^2+1)}\right] + \left[\frac{-\frac{1}{4}(s+3)}{(s+p_1)(s+p_2)}\right]\left[\frac{\text{num2}}{s(s^2+104)}\right]$$

$$= \frac{A}{s} + \frac{B}{s+p_1} + \frac{C}{s+p_2} + \frac{D}{s-j1} + * + \frac{E}{s} + \frac{F}{s+p_1} + \frac{G}{s+p_2} + \frac{H}{s-j100} + *$$

$y = y_1 + y_2$

$B = \lim_{s \to -p_1} (s+p_1) Y_1(s)$

$C = \lim_{s \to -p_2} (s+p_2) Y_1(s)$

$F = \lim_{s \to -p_1} (s+p_1) Y_2(s)$

$G = \lim_{s \to -p_2} (s+p_2) Y_2(s)$

$y_1 = A + Be^{-p_1 t} + Ce^{-p_2 t} + 2|D| \cos(t + \angle D)$

$y_2 = E + Fe^{-p_1 t} + Ge^{-p_2 t} + 2|H| \cos(100t + \angle H)$

y_{2ss}

$-2|D| \cos(t + \angle D - 180°) = -2|D| \sin(t + \angle D - 180° + 90°)$

y_{1ss}

$A = \lim_{s \to 0} s Y_1(s) = 10 H_1(0)$

$D = \lim_{s \to j1} (s - j1) Y_1(s) = \frac{|H_1(j1)|}{2} e^{j(20° + \angle H_1(j1) + 180 - 90°)}$

$E = \lim_{s \to 0} s Y_2(s) = -3 H_2(0)$

$H = \lim_{s \to j100} (s - j100) Y_2(s) = \frac{|H_2(j100)|}{2} e^{j(-90° + \angle H_2(j100))}$

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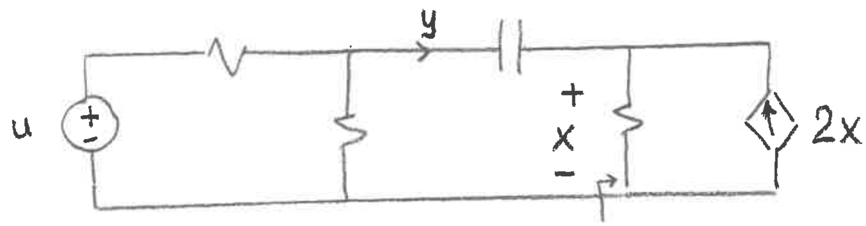
Problem #4

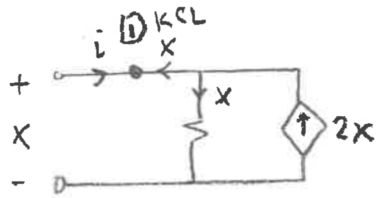
Determine H, diff eq, y, y_{ss} (All R, C = 1)



Hint: Find R_{th}

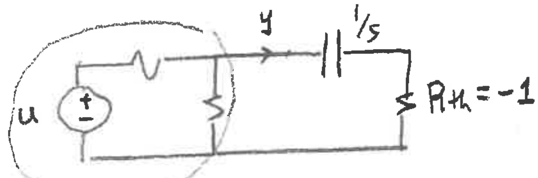
$u = 4 + \sin 2t$

solution using hint (finding R_{th})

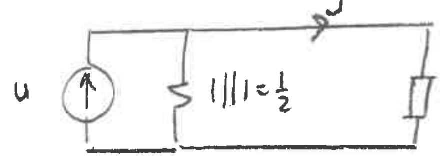


$\Rightarrow$ ① KCL: $i = -x$

$x = -i \Rightarrow \boxed{R_{th} = -1}$



source transf



series

$-1 + \frac{1}{s} = \frac{1-s}{s}$

Current Division:

$$y = \left[\frac{\frac{1}{2}}{\frac{1}{2} + \frac{1-s}{s}}\right] u = \left[\frac{\frac{1}{2}s}{\frac{1}{2}s + 1 - s}\right] u$$

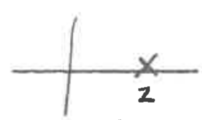
$1 - \frac{1}{2}s$

$$= \left[\frac{s}{2-s}\right] u$$

$$\boxed{H = \frac{y}{u} = \frac{-s}{s-2}}$$

$\hookrightarrow \dot{y} - 2y = -\dot{u}$

$\Phi = s - 2 = 0 \Rightarrow s = 2$



pole in RHP $\Rightarrow$ unstable system

MOTF:
component of y that can be computed using MOTF:

$$y_{MOTF} = 4H(0) + |H(j2)| \sin(2t + \angle H(j2))$$

$H(0) = 0$

$$H(j2) = \frac{-j2}{j2-2} = \frac{2e^{j(-90^\circ)}}{2\sqrt{2}\, e^{j135^\circ}}$$

$$= \frac{1}{\sqrt{2}}\, e^{j(-90-135^\circ)} \quad -225^\circ$$

$\angle H(j2)$

$|H(j2)|$

Note: The following is also correct:

$$H(j2) = \frac{j1}{1-j1} = \frac{1e^{j90^\circ}}{\sqrt{2}e^{-j45}} = \frac{1}{\sqrt{2}} e^{j(90^\circ+45^\circ)} \quad 135^\circ$$

num

$$U(s) = \frac{4}{s} + \frac{2}{s^2+4} = \frac{4s^2+2s+16}{s(s^2+4)}$$

--- lets now find $y(t)$ ---

Problem #4

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$$Y(s) = H(s)U(s) = \left[\frac{-s}{s-2}\right]\left[\frac{\text{num}}{s(s^2+4)}\right] = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s-j2} + *$$

$H(s)$; num $= 4s^2+2s+16$

$$\Rightarrow \; y = A + Be^{2t} + 2|C|\cos(2t + \angle C)$$

$2|C|\sin(2t+\angle C + 90^\circ)$

$\uparrow$ unstable y_{ss}

y_{MOTF}

$$B = \lim_{s\to 2}(s-2)Y(s)$$

$$= -s\left[\frac{4s^2+2s+16}{s(s^2+4)}\right]\Bigg|_{s=2}$$

$$= -\cancel{2}\left[\frac{4(4)-4+16}{(-\cancel{2})(4+4)}\right]$$

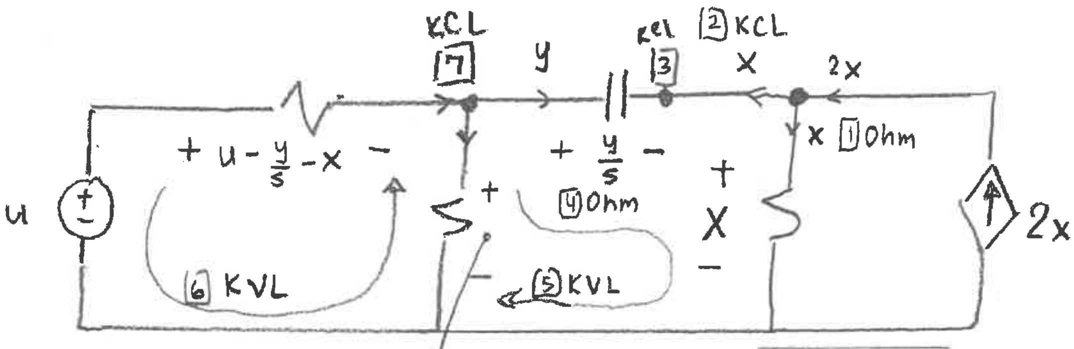
$$= \frac{28}{8} = \frac{7}{2}$$

$$\boxed{B = 3.5}$$

$$A = \lim_{s\to 0} s\,Y(s) = 4\,H(0)$$

$$C = \lim_{s\to j2}(s-j2)Y(s) = \frac{|H(j2)|}{2}e^{j(\angle H(j2) - 90^\circ)}$$

Solution via KVL, KCL, Ohm



[3] KCL = $\boxed{y = -x}$ $\Rightarrow$ $x = -y$

[5] $\frac{y}{s} + x$

[7] KCL = $\boxed{\left(u - \frac{y}{s} - x\right) = \left(\frac{y}{s} + x\right) + y}$

$$\Longrightarrow u = \frac{y}{s} + x + \frac{y}{s} + x + y \quad (x = -y)$$

$$= y\left[\frac{1}{s} - 1 + \frac{1}{s} \cancel{-1} \cancel{+1}\right]$$

$$= y\left[\frac{2}{s} - 1\right]$$

$$= y\left[\frac{2-s}{s}\right]$$

$$\Rightarrow \boxed{H = \frac{y}{u} = \frac{-s}{s-2}}$$

as obtained earlier!

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Problem #4

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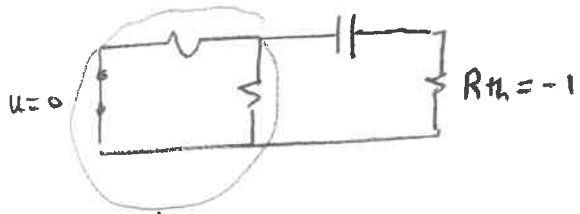
pole check: (use $R_{th} = -1$)

set $u = 0$



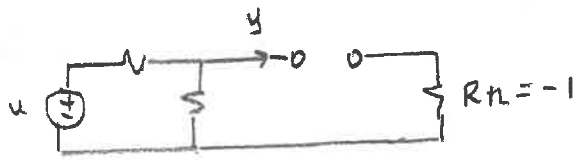
$1 \| 1 = \frac{1}{2} \Rightarrow Z = \frac{1}{2} + \frac{1}{s} - 1 = \frac{1}{s} - \frac{1}{2} = \frac{1 - \frac{1}{2}s}{s} = \frac{-\frac{1}{2}\underbrace{(s-2)}_{\Phi}}{s}$

$\Rightarrow \Phi = s - 2$

as we obtained earlier!

Gain Checks: (I'll use $R_{th} = -1$)

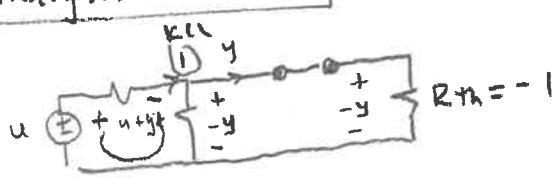
$\boxed{\text{Analysis at } s = 0}$ $Z_C = \frac{1}{sC} = \infty$ (open)



$\Rightarrow y = 0 = 0u$

$\Rightarrow H(0) = 0$ ← Agrees with our $H(s) = \frac{-s}{s-2}$

$\boxed{\text{Analysis at } s = \infty}$ $Z_C = \frac{1}{sC} = 0$ (short)



1 KCL:

$\overrightarrow{u+y} = (\downarrow{-y}) + \overrightarrow{y}$

$\Rightarrow y = -u$

$\Rightarrow H(\infty) = -1$ ⇐ Agrees with

– IT'S ALWAYS GOOD TO PERFORM THE ABOVE CHECKS!

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Note: Show how to Compute all coefficients associated with constants & sinusoids & instabilities in y

Problem #5

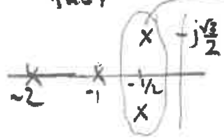
Determine t_s, y_{ss}, y

$$H(s) = \frac{(s+0.01)(s^2+4)(s^2+64)}{(s+1)(s+2)(s^2+s+1)(s^2-4s+8)}$$

$u = -10 + 2\cos(0.01t - 45°) + 3\sin(2t+20°) - 4\sin(8t-24°) + 5\cos(100t+90°)$

solution:

stable fast



$-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ stable (slow poles)

$t_s = \frac{5}{\frac{1}{2}} = 10\,sec$

$2 \pm j2$ unstable!

component of y that can be computed using MOTF!

$$y_{MOTF} = -10H(0) + 2|H(j0.01)|\cos(0.01t - 45° + \angle H(j0.01))$$
$$+ 3|H(j2)|\sin(2t + 20° + \angle H(j2))$$
$$- 4|H(j8)|\sin(8t - 24° + \angle H(j8))$$
$$+ 5|H(j100)|\cos(100t + 90° + \angle H(j100))$$

zero since $H(j2) = 0$

zero since $H(j8) = 0$

$$H(0) = \frac{(0.01)(4)(64)}{(1)(2)(1)(8)}$$

$$H(j0.01) \cong \left. \frac{(s+0.01)\overbrace{(4)(64)}^{M}}{(1)(2)(1)(8)} \right|_{s=j0.01} = M(0.01)(1+j1)$$

$$= \underbrace{0.01M\sqrt{2}}_{|H(j0.01)|}\, e^{j\overbrace{45°}^{\angle H(j0.01)}}$$

$H(j2) = H(j8) = 0$ as stated above

$$H(j100) \cong \left. \frac{s^5}{s^6} \right|_{s=j100} = \left. \frac{1}{s} \right|_{s=100e^{j90°}} = \frac{1}{100e^{j90°}} = \underbrace{0.01}_{|H(j100)|}\, e^{j\overbrace{(-90°)}^{\angle H(j100)}}$$

--- Now lets find y (the form of y!!!)

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$\boxed{\text{Problem \# 5}}$ EEE 202 Fall 2024 Exam #2 Fri 12-6-24

$$U(s) = \frac{-10}{s} + \frac{\quad}{s^2+10^{-4}} + \frac{\quad}{s^2+4} + \frac{\quad}{s^2+64} + \frac{\quad}{s^2+10^4}$$

$$= \frac{\text{num}}{s(s^2+10^{-4})(s^2+4)(s^2+64)(s^2+10^4)}$$

$$Y(s) = H(s)U(s) = \left[\frac{(s+0.01)\cancel{(s^2+4)(s^2+64)}}{(s+1)(s+2)\underbrace{(s^2+s+1)}_{-\frac{1}{2}\pm j\frac{\sqrt{3}}{2}}\underbrace{(s^2-4s+8)}_{2\pm j2}}\right]\left[\frac{\text{num}}{s\underbrace{(s^2+10^{-4})}_{\pm j10^{-2}}\cancel{(s^2+4)(s^2+64)}\underbrace{(s^2+10^4)}_{\pm j100}}\right]$$

$$= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} + \left(\frac{D}{s+\frac{1}{2}-j\frac{\sqrt{3}}{2}} + *\right) + \left(\frac{E}{s-2-j2} + *\right) + \left(\frac{F}{s-j0.01} + *\right)$$

$$+ \left(\frac{G}{s-j100} + *\right)$$

stable, $t_s = \frac{5}{\frac{1}{2}} = 10\,\text{sec}$

$$\Rightarrow y = A + Be^{-t} + Ce^{-2t} + 2|D|e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t + \angle D\right)$$

$$+ 2|E|e^{2t}\cos(2t + \angle E)$$ — unstable, y_{ss} !

$$+ 2|F|\cos(0.01t + \angle F)$$

$$+ 2|G|\cos(100t + \angle G)$$

Non-zero components of y_{HOTF} computed earlier!

$$A = \lim_{s\to 0} sY(s) = -10\,H(0)$$

$$F = \lim_{s\to j0.01} (s-j0.01)Y(s) = |H(j0.01)|\,e^{j\left(-45^\circ + \underline{\angle H(j0.01)}\right)}$$

$$G = \lim_{s\to j100} (s-j100)Y(s) = \frac{5|H(j100)|}{2}\,e^{j\left(90^\circ + \underline{\angle H(j100)}\right)}$$

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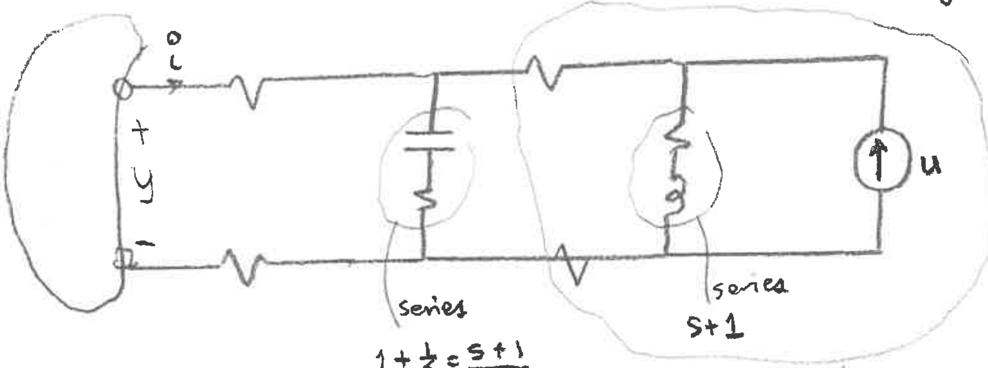
Problem # 6

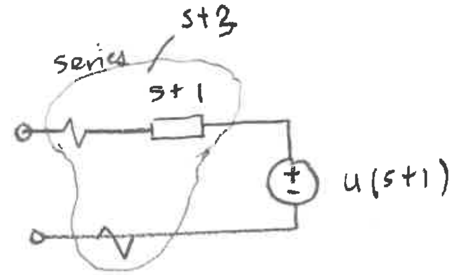
Determine an s-domain Thevenin equivalent at y (All R, L, C = 1)

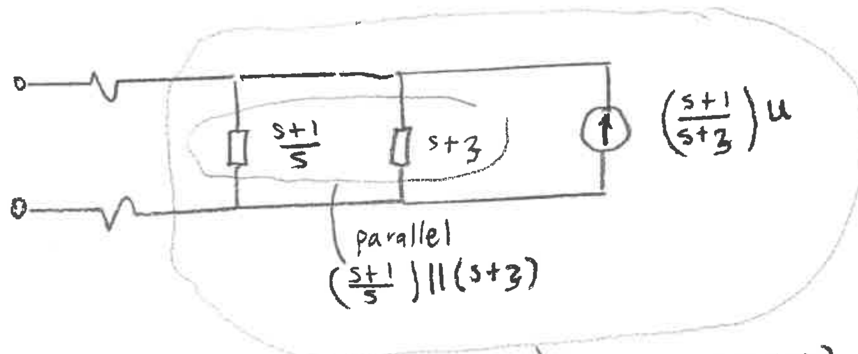
(specify Zth & Vth ... emphasize process ... not algebra!)

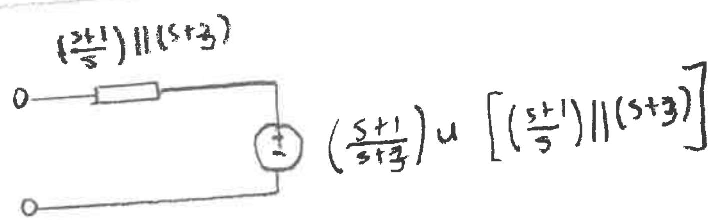


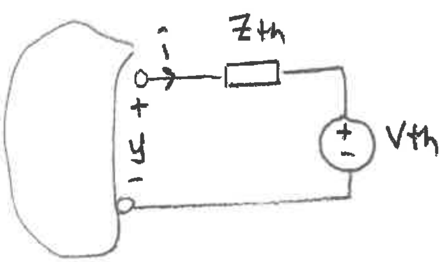
solution:

lets use source transformations:









$$Z_{th} = 2 + \left[\left(\frac{s+1}{s}\right)\|(s+3)\right]$$

$$V_{th} = \left[\left(\frac{s+1}{s}\right)\|(s+3)\right]\left(\frac{s+1}{s+3}\right)u$$

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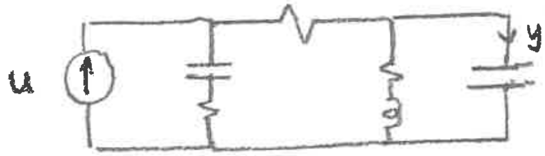
Problem #7

Determine H, y_{ss}, t_s

Compute all coefficients in y_{ss} (approximately)!

(all $R, L, C = 1$)

Hint: poles = -0.3522, $-0.8239 \pm j0.8607$



$u = -7 + 100 \sin(0.01t - 90°) - 2\cos(100t - 5°)$

solution:

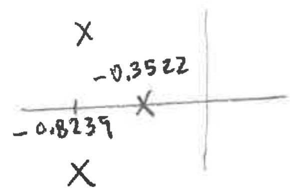
system is stable since it only has R, L, Cs & an indep source

(a dependent source is necessary (not sufficient) to get an unstable system!)

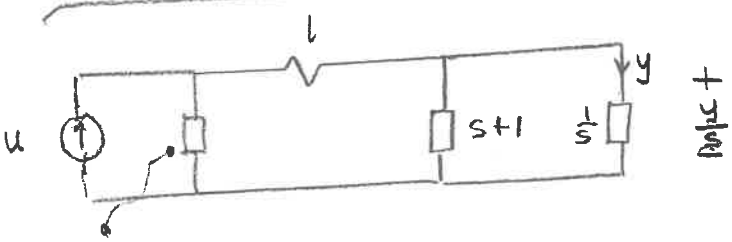
$$\Longrightarrow \quad y_{ss} \overset{\text{HOTF}}{=} -7H(0) + 100\,|H(j0.01)|\sin(0.01t - 90° + \angle H(j0.01))$$

$$-2\,|H(j100)|\cos(100t - 5° + \angle H(j100))$$

$$t_s = \frac{5}{|\text{slow pole}|} = \frac{5}{0.3522}$$

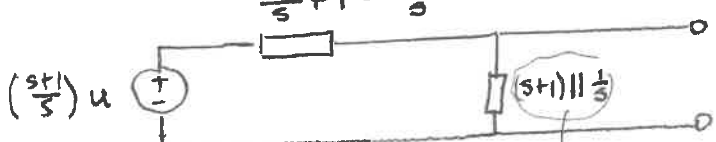


Lets find H via source transformations:



$\frac{1}{s} + 1 = \frac{s+1}{s}$

$\frac{s+1}{s} + 1 = \frac{2s+1}{s}$



$$\frac{\frac{s+1}{s}}{s+1+\frac{1}{s}} = \frac{s+1}{s^2+s+1}$$

voltage division

$$+\ y/s\ - \overset{\text{voltage division}}{=} \left[\frac{\frac{s+1}{s^2+s+1}}{\left(\frac{2s+1}{s}\right) + \left(\frac{s+1}{s^2+s+1}\right)}\right]\left(\frac{s+1}{s}\right)u$$

$$= \left[\frac{\cancel{s}(s+1)}{(2s+1)(s^2+s+1) + s(s+1)}\right]\left(\frac{s+1}{\cancel{s}}\right)u$$

$2s^3 + 2s^2 + 2s + s^2 + s + 1 + s^2 + s$

$2s^3 + 4s^2 + 4s + 1$

$= 2(s^3 + 2s^2 + 2s + \frac{1}{2})$

$$\Longrightarrow \quad \frac{y}{s} = \left[\frac{\frac{1}{2}(s+1)^2}{s^3 + 2s^2 + 2s + \frac{1}{2}}\right]u$$

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$\boxed{\text{Problem \# 7}}$ EEE 202 Fall 2024 Exam #2 Fri 12-6-24

$$H = \frac{y}{u} = \frac{\frac{1}{2} s (s+1)^2}{s^3 + 2s^2 + 2s + \frac{1}{2}}$$

Note: roots are -0.3522, $-0.8239 \pm j0.8607$

– via calculator or MATLAB!

Compute gains needed in y_{ss}

$H(0) = 0$

$$H(j0.01) \cong \left. \frac{\cancel{\tfrac{1}{2}}\, s}{\cancel{\tfrac{1}{2}}} \right|_{s = 10^{-2} e^{j90^\circ}} = 10^{-2} e^{j90^\circ}$$

$\sim |H(j0.01)|$; $\sim \angle H(j0.01)$

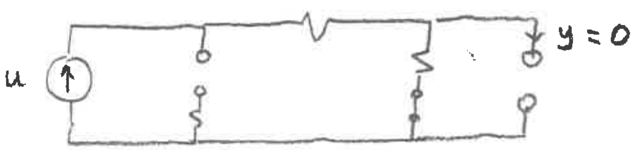
$$H(j100) \cong \frac{1}{2} = \frac{1}{2} e^{j0^\circ}$$

$\sim \angle H(j100)$; $\sim |H(j100)|$

Gain Checks on H =

$\boxed{\text{Analysis at } s = 0}$

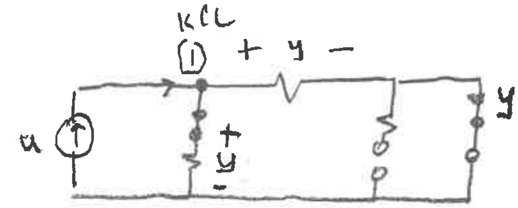
$Z_L = sL = 0$ (short)
$Z_C = \frac{1}{sC} = \infty$ (open)



$y = 0 \Rightarrow H(0) = 0$ as expected!

$\boxed{\text{Analysis at } s = \infty}$

$Z_L = sL = \infty$ (open)
$Z_C = \frac{1}{sC} = 0$ (short)



(1) KCL: $u = y + y = 2y$

$\Rightarrow y = \frac{1}{2} u$

$\Rightarrow H(\infty) = \frac{1}{2}$

as expected!

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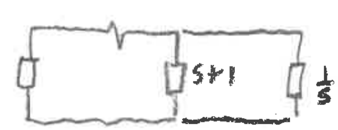
Characteristic Eq Check: (set $u = 0$)

$\frac{1}{s} + 1 = \frac{s+1}{s}$



$\Rightarrow$

$\frac{s+1}{s} + 1 = \frac{2s+1}{s}$

$$(s+1) \parallel \left(\frac{1}{s}\right) = \frac{\frac{s+1}{s}}{s+1+\frac{1}{s}} = \frac{s+1}{s^2+s+1}$$

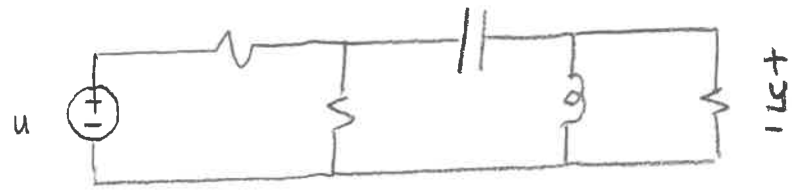
$$Z = \frac{2s+1}{s} + \frac{s+1}{s^2+s+1} = \frac{(2s+1)(s^2+s+1) + s(s+1)}{s(s^2+s+1)}$$

$\Rightarrow$ Numerator yields Φ obtained earlier!

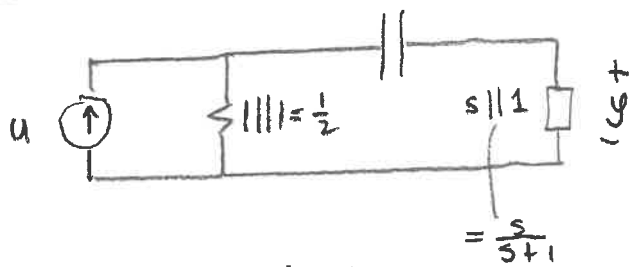
EEE 202 Fall 2024 Exam #2 Fri 12-6-24

Problem # 8

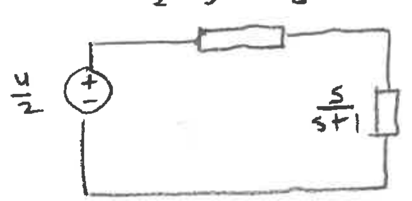
Determine & plot $|H(j\omega)|$ & $\angle H(j\omega)$ (All R, L, C = 1)



solution via source transformations:



$\frac{1}{2}+\frac{1}{s}=\frac{\frac{1}{2}s+1}{s}$



$+\;y\;-$ $\overset{\text{voltage divider}}{=} \left[\dfrac{\frac{s}{s+1}}{\left(\frac{\frac{1}{2}s+1}{s}\right)+\frac{s}{s+1}}\right]\dfrac{u}{2}$

$$H=\frac{y}{u}=\frac{\frac{1}{2}s^2}{\left(\frac{1}{2}s+1\right)(s+1)+s^2}$$

$$\frac{1}{2}s^2+\frac{1}{2}s+s+1+s^2$$

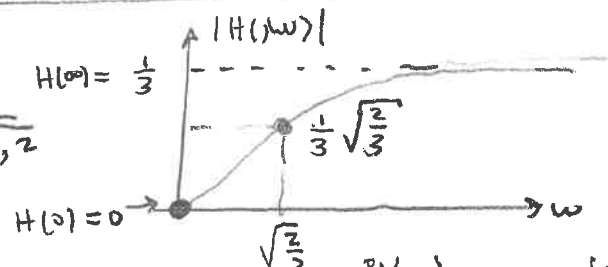
$$=\frac{3}{2}s^2+\frac{3}{2}s+1$$

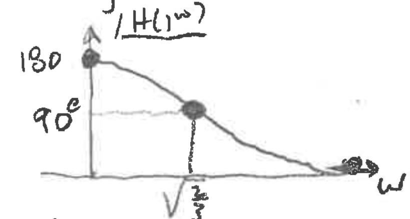
$$=\frac{3}{2}\left[s^2+s+\frac{2}{3}\right]$$

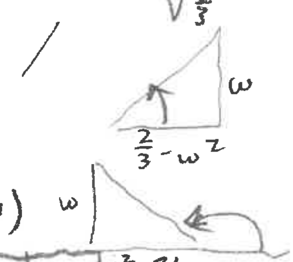
$$\boxed{H=\frac{\frac{1}{3}s^2}{s^2+s+\frac{2}{3}}}$$

$$H(j\omega)=\frac{-\frac{1}{3}\omega^2}{\frac{2}{3}-\omega^2+j\omega}$$

$$|H(j\omega)|=\frac{\frac{1}{3}\omega^2}{\sqrt{\left(\frac{2}{3}-\omega^2\right)^2+\omega^2}}$$







$$\angle H(j\omega)=\angle top-\angle bottom=180-\begin{cases}\tan^{-1}\left(\frac{\omega}{\frac{2}{3}-\omega^2}\right) & \omega\in\left[0,\sqrt{\frac{2}{3}}\right]\\ 180^\circ-\tan^{-1}\left(\frac{\omega}{\omega^2-\frac{2}{3}}\right) & \omega\in\left[\sqrt{\frac{2}{3}},\infty\right)\end{cases}$$

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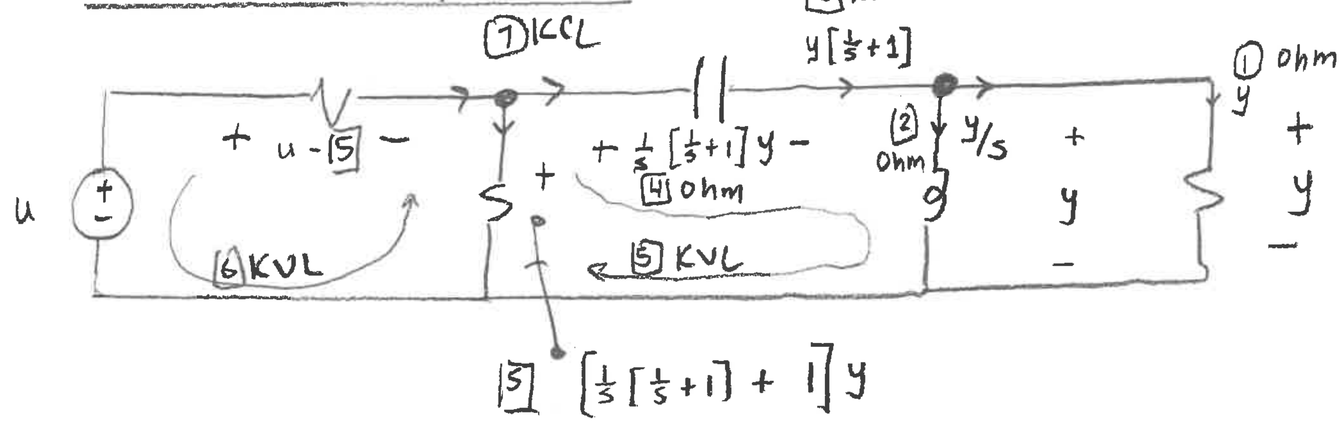
Problem #8

solution via KVL, KCL, Ohm



$\boxed{7}$ KCL:
$$\boxed{\left(\frac{u - \left[\frac{1}{s}\left(\frac{1}{s}+1\right)+1\right]y}{1}\right) = \frac{\left(\frac{1}{s}\left(\frac{1}{s}+1\right)+1\right)y}{1} + y\left(\frac{1}{s}+1\right)}$$

$$\Longrightarrow y\left[\frac{1}{s}\left(\frac{1}{s}+1\right)+1 + \frac{1}{s}\left(\frac{1}{s}+1\right)+1 + \frac{1}{s}+1\right] = u$$

$$\Longrightarrow y\left[\frac{2}{s^2} + \frac{3}{s} + 3\right] = u$$

$$\Longrightarrow y\left[\underbrace{2 + 3s + 3s^2}_{3\left(s^2+s+\frac{2}{3}\right)}\right] = s^2 u$$

$$\Longrightarrow \boxed{H = \frac{y}{u} = \frac{\frac{1}{3}s^2}{s^2+s+\frac{2}{3}}}$$

which agrees with what we obtained earlier

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